

ARE THERE STERILE NEUTRINOS IN THE FLUX OF SOLAR NEUTRINOS ON THE EARTH?

S.M. Bilenky*

*INFN Torino, Via P. Giuria 1, 10125 Torino, Italy
Dipartimento di Fisica Teorica, Università di Torino
and
Joint Institute for Nuclear Research, Dubna, Russia*

C. Giunti†

*INFN Torino, Via P. Giuria 1, 10125 Torino, Italy
Dipartimento di Fisica Teorica, Università di Torino
(February 1994)*

Abstract

It is shown that the future SNO and Super-Kamiokande experiments, in which high energy ^8B neutrinos will be detected through the observation of CC, NC and ν - e elastic scattering processes, could allow to reveal in a model independent way the presence of sterile neutrinos in the flux of solar neutrinos on the earth. Lower bounds for different averaged values of the probability of transition of solar ν_e 's into sterile states and for the total flux of ^8B neutrinos are derived in terms of measurable quantities. The possibilities to reveal the presence of ν_μ and/or ν_τ in the solar neutrino flux on the earth are also considered and the case of transitions of solar ν_e 's only into sterile states is discussed. Some numerical results for a simple model with ν_e - ν_s mixing are given.

Typeset using REVTeX

*E-mail address: BILENKY@TO.INFN.IT.

†E-mail address: GIUNTI@TO.INFN.IT.

I. INTRODUCTION

The problem of neutrino masses and mixing is the most actual one in present day neutrino physics. As it was first pointed out by B. Pontecorvo [1], solar neutrino experiments are very important tools for the investigation of this problem. At present four solar neutrino experiments (Homestake [2], Kamiokande [3], GALLEX [4] and SAGE [5]) investigate different ranges of the solar neutrino spectrum. The observed event rates in all four experiments are significantly smaller than the event rates predicted by the Standard Solar Model (SSM) [6–8]. Phenomenological analyses of the existing data [9] give some indications in favor of a non-astrophysical explanation of the observed "deficit" of solar neutrinos and it was shown [10] that all data are well described in the simplest case of mixing between ν_e and ν_μ (or ν_τ) if MSW resonant transitions in matter [11] take place. For the two parameters Δm^2 and $\sin^2 2\theta$ ($\Delta m^2 \equiv m_2^2 - m_1^2$, m_1 and m_2 are the neutrino masses, and θ is the mixing angle) the following values were obtained:

$$\Delta m^2 \simeq 5 \times 10^{-6} \text{ eV}^2 \quad \text{and} \quad \sin^2 2\theta \simeq 8 \times 10^{-3} \quad (\text{small mixing angle solution}) , \quad (1.1)$$

$$\Delta m^2 \simeq 10^{-5} \text{ eV}^2 \quad \text{and} \quad \sin^2 2\theta \simeq 0.8 \quad (\text{large mixing angle solution}) . \quad (1.2)$$

The data can also be described by ν_e and ν_μ (or ν_τ) vacuum oscillations [12] if the parameter Δm^2 has such a small value that the term $\cos\left(\frac{\Delta m^2 R}{2E}\right)$ (R is the sun-earth distance and E is the neutrino energy) in the expression for the probability of vacuum transitions does not disappear due to averaging (i.e. the oscillation length is of the same order of magnitude as the distance between the sun and the earth). In this case, for the two parameters Δm^2 and $\sin^2 2\theta$ the following values were found:

$$\Delta m^2 \simeq 8 \times 10^{-11} \text{ eV}^2 \quad \text{and} \quad \sin^2 2\theta \simeq 0.8 . \quad (1.3)$$

Finally, all the existing data can also be described [13] by MSW transitions of solar ν_e 's into sterile states (see, for example Ref. [14]). In this case only the small mixing angle solution is allowed¹:

$$\Delta m^2 \simeq 5 \times 10^{-6} \text{ eV}^2 \quad \text{and} \quad \sin^2 2\theta \simeq 7 \times 10^{-3} \quad (\text{small mixing angle solution}) . \quad (1.4)$$

This solution is compatible with the constraints on the ν_e - ν_s mixing parameters obtained from big bang nucleosynthesis [15].

Thus, even if we assume that the SSM correctly predicts the values of the neutrino fluxes from all the main reactions, the existing solar neutrino data do not allow to determine in which neutrino states (active or sterile) solar ν_e 's are transformed.

The knowledge of the neutrino states in which solar neutrinos are transferred is of fundamental importance for the theory. Transitions of solar ν_e 's into active ν_μ and/or ν_τ are possible in many models beyond the Standard Model (see, for example, Ref. [16]). Such

¹ Let us stress that the solutions (1.1)–(1.4) were obtained under the assumption that the values of the neutrino fluxes are given by the SSM.

transitions are also possible in the Standard Model if the neutrino masses and mixing are generated by the same standard Higgs mechanism which generates the masses and mixing of quarks and the masses of the charged leptons. On the other hand, transitions of solar ν_e 's into sterile states are possible only in the models beyond the Standard Model. Thus, a discovery of $\nu_e \rightarrow \nu_s$ transitions would be a clear signal of new physics beyond the Standard Model.

In Ref. [17] we discussed the possibilities of the solar neutrino experiments of the next generation (SNO [18], Super-Kamiokande [19]), in which high energy ^8B neutrinos will be detected, to obtain informations on the transitions of solar ν_e 's into sterile states. Here we will derive several relations which could allow to reveal the presence of sterile neutrinos in the solar neutrino flux on the earth independently on any assumption about the value of the initial ^8B neutrino flux. We will also show that from the data of the SNO and S-K experiments it will be possible to obtain a model independent lower bound for the initial ^8B ν_e flux in the general case of transitions of ν_e 's into active and sterile neutrinos. A comparison of this lower bound with the value of the ^8B neutrino flux predicted by the SSM will be a direct experimental test of the model which does not depend on possible transitions of solar ν_e 's into other states. Furthermore, we will discuss the possibilities of SNO and S-K to reveal the presence of ν_μ and/or ν_τ in the solar neutrino flux on the earth and we will consider the special case of transitions of solar ν_e 's only into sterile states. Finally, we will present some numerical results obtained in a simple model with ν_e - ν_s mixing.

In the SNO experiment (scheduled to start in the end of 1995) solar neutrinos will be detected through the observation of *three* different processes:

1. The CC process

$$\nu_e + d \rightarrow e^- + p + p ; \quad (1.5)$$

2. The NC process

$$\nu + d \rightarrow \nu + p + n ; \quad (1.6)$$

3. The CC+NC elastic scattering (ES) process

$$\nu + e^- \rightarrow \nu + e^- . \quad (1.7)$$

In the CC process (1.5) the electron spectrum will be measured and the flux of solar ν_e 's on the earth as a function of neutrino energy E , $\phi_{\nu_e}(E)$, will be determined [18]. The NC process (1.6) will be detected through the observation of neutrons. Only the total number of NC events will be measured.

In the S-K experiment (scheduled to start in 1996) solar neutrinos will be detected through the observation of the ES process (1.7). The event rate is expected to be about 50 times larger than in the current Kamiokande III experiment and the spectrum of the recoil electrons will be measured with high accuracy [19].

In both the SNO and S-K experiments, due to the high energy thresholds ($\simeq 6$ MeV for the CC process, 2.2 MeV for the NC process and $\simeq 5$ MeV for the ES process), only

neutrinos coming from ${}^8\text{B}$ decay will be detected. The energy spectrum of the initial ${}^8\text{B}$ ν_e 's can be written as

$$\phi_B(E) = \Phi_B X(E) . \quad (1.8)$$

Here $X(E)$ is a known normalized function determined by the phase space factor² of the decay ${}^8\text{B} \rightarrow {}^8\text{Be} + e^+ + \nu_e$, (the small corrections due to forbidden transitions were calculated in Ref. [21]) and Φ_B is the total flux of initial ${}^8\text{B}$ solar ν_e 's.

II. LOWER BOUND FOR THE ${}^8\text{B}$ NEUTRINO FLUX

In this section we will show that from the SNO and S-K data it will be possible to obtain a model independent *lower bound* for the ${}^8\text{B}$ neutrino flux Φ_B in the general case of transitions of solar ν_e 's into active and sterile neutrinos. As we will see in the next section, the knowledge of a lower bound for the ${}^8\text{B}$ neutrino flux will allow to obtain upper bounds for different averaged values of the probability of transition of solar ν_e 's into ν_e , ν_μ and ν_τ . If it will occur that any of these upper bounds is less than one, it will mean that there are sterile neutrinos in the flux of solar neutrinos on the earth.

Let us consider first the NC process (1.6). Using the ν_e - ν_μ - ν_τ universality of NC, we have

$$\left\langle \sum_{\ell=e,\mu,\tau} P_{\nu_e \rightarrow \nu_\ell} \right\rangle_{\text{NC}} = \frac{N^{\text{NC}}}{X_{\nu d}^{\text{NC}} \Phi_B} . \quad (2.1)$$

Here N^{NC} is the total NC event rate, $P_{\nu_e \rightarrow \nu_\ell}$ is the probability of transition of solar ν_e 's into ν_ℓ (with $\ell = e, \mu, \tau$) and

$$X_{\nu d}^{\text{NC}} \equiv \int_{E_{\text{th}}^{\text{NC}}} \sigma_{\nu d}^{\text{NC}}(E) X(E) dE , \quad (2.2)$$

where $\sigma_{\nu d}^{\text{NC}}(E)$ is the cross section for the process $\nu d \rightarrow \nu n p$ and $E_{\text{th}}^{\text{NC}}$ is the threshold neutrino energy. Using the results of a recent calculation of the cross-section $\sigma_{\nu d}^{\text{NC}}(E)$ [22] we obtained $X_{\nu d}^{\text{NC}} = 4.72 \times 10^{-43} \text{ cm}^2$. The average probability $\left\langle \sum_{\ell=e,\mu,\tau} P_{\nu_e \rightarrow \nu_\ell} \right\rangle_{\text{NC}}$ is determined as follows:

$$\left\langle \sum_{\ell=e,\mu,\tau} P_{\nu_e \rightarrow \nu_\ell} \right\rangle_{\text{NC}} \equiv \frac{1}{X_{\nu d}^{\text{NC}}} \int_{E_{\text{th}}^{\text{NC}}} \sigma_{\nu d}^{\text{NC}}(E) X(E) \sum_{\ell=e,\mu,\tau} P_{\nu_e \rightarrow \nu_\ell}(E) dE . \quad (2.3)$$

² It was shown in Ref. [20] that the distortions of the neutrino spectra are negligibly small under solar conditions.

Taking into account that $\left\langle \sum_{\ell=e,\mu,\tau} P_{\nu_e \rightarrow \nu_\ell} \right\rangle_{\text{NC}} \leq 1$, from Eq.(2.1) we obtain the following lower bound for the total flux of ^8B neutrinos:

$$\Phi_B \geq \frac{N^{\text{NC}}}{X_{\nu d}^{\text{NC}}} . \quad (2.4)$$

Another lower bound for the flux Φ_B can be obtained from the CC data. The ν_e survival probability is connected with the flux of ν_e on the earth by the relation

$$P_{\nu_e \rightarrow \nu_e}(E) = \frac{\phi_{\nu_e}(E)}{X(E) \Phi_B} . \quad (2.5)$$

From this relation it follows that

$$\Phi_B \geq \left[\frac{\phi_{\nu_e}(E)}{X(E)} \right]_{\text{max}} , \quad (2.6)$$

where $[\phi_{\nu_e}(E)/X(E)]_{\text{max}}$ is the maximal value of the function $\phi_{\nu_e}(E)/X(E)$ in the explored energy range.

Analogously, we have

$$\Phi_B \geq \frac{N^{\text{CC}}}{X_{\nu d}^{\text{CC}}} . \quad (2.7)$$

Here N^{CC} is the total CC event rate and

$$X_{\nu d}^{\text{CC}} \equiv \int_{E_{\text{th}}^{\text{CC}}} \sigma_{\nu d}^{\text{CC}}(E) X(E) dE , \quad (2.8)$$

where $\sigma_{\nu d}^{\text{CC}}(E)$ is the cross section for the process $\nu_e d \rightarrow e^- p p$. Using the results of Ref. [22] we obtained $X_{\nu d}^{\text{CC}} \simeq 1.1 \times 10^{-42} \text{ cm}^2$ for $E_{\text{th}}^{\text{CC}} \simeq 6 \text{ MeV}$.

The total number of ES events N^{ES} is given by

$$N^{\text{ES}} = \int_{E_{\text{th}}^{\text{ES}}} \sum_{\ell=e,\mu,\tau} \sigma_{\nu_\ell e}(E) P_{\nu_e \rightarrow \nu_\ell}(E) X(E) dE \Phi_B , \quad (2.9)$$

where $\sigma_{\nu_\ell e}(E)$ is the total cross section of the process $\nu_\ell e \rightarrow \nu_\ell e$ (with $\ell = e, \mu, \tau$). Taking into account that $\sigma_{\nu_\mu e}(E)/\sigma_{\nu_e e}(E) \simeq 1/6$, from Eq.(2.9) we have

$$N^{\text{ES}} \leq \int_{E_{\text{th}}^{\text{ES}}} \sigma_{\nu_e e}(E) \sum_{\ell=e,\mu,\tau} P_{\nu_e \rightarrow \nu_\ell}(E) X(E) dE \Phi_B , \quad (2.10)$$

From Eq.(2.10) we obtain the following lower bound for Φ_B

$$\Phi_B \geq \frac{N^{\text{ES}}}{X_{\nu_e e}} , \quad (2.11)$$

where

$$X_{\nu_e e} \equiv \int_{E_{\text{th}}^{\text{ES}}} \sigma_{\nu_e e}(E) X(E) dE \quad (2.12)$$

For $E_{\text{th}}^{\text{ES}} \simeq 5 \text{ MeV}$ we have $X_{\nu_e e} \simeq 2 \times 10^{-44} \text{ cm}^2$. The values of N^{ES} and $X_{\nu_e e}$ depend on the threshold energy $E_{\text{th}}^{\text{ES}}$. Let us notice that it is worthwhile to study the dependence of $N^{\text{ES}}/X_{\nu_e e}$ on the threshold energy $E_{\text{th}}^{\text{ES}}$ in order to choose the optimal value of this quantity (providing that the statistical accuracy is sufficiently high).

In the S-K experiment the spectrum of ES recoil electrons $n^{\text{ES}}(\text{T})$ (T is the electron kinetic energy) will be measured with high accuracy. We have

$$n^{\text{ES}}(\text{T}) = \int_{E_{\text{m}}(\text{T})} \sum_{\ell=e,\mu,\tau} \frac{d\sigma_{\nu_\ell e}}{dT}(E, \text{T}) P_{\nu_e \rightarrow \nu_\ell}(E) X(E) dE \Phi_{\text{B}}. \quad (2.13)$$

Here $\frac{d\sigma_{\nu_\ell e}}{dT}(E, \text{T})$ is the differential cross section of the process $\nu_\ell e \rightarrow \nu_\ell e$ (with $\ell = e, \mu, \tau$) and $E_{\text{m}}(\text{T}) = \frac{1}{2} \text{T} \left(1 + \sqrt{1 + 2 m_e/\text{T}}\right)$. From Eq.(2.13) it follows that

$$n^{\text{ES}}(\text{T}) \leq \int_{E_{\text{m}}(\text{T})} \frac{d\sigma_{\nu_e e}}{dT}(E, \text{T}) \sum_{\ell=e,\mu,\tau} P_{\nu_e \rightarrow \nu_\ell}(E) X(E) dE \Phi_{\text{B}}. \quad (2.14)$$

From Eq.(2.14) we obtain the following lower bound for the total ^8B neutrino flux:

$$\Phi_{\text{B}} \geq \left[\frac{n^{\text{ES}}(\text{T})}{X_{\nu_e e}(\text{T})} \right]_{\text{max}}, \quad (2.15)$$

where

$$X_{\nu_e e}(\text{T}) \equiv \int_{E_{\text{m}}(\text{T})} \frac{d\sigma_{\nu_e e}}{dT}(E, \text{T}) X(E) dE. \quad (2.16)$$

The function $X_{\nu_e e}(\text{T})$ is plotted in Fig.1.

It is clear from the derivation of Eqs.(2.11) and (2.15) that, if an appreciable part of solar ν_e 's are transformed into ν_μ and/or ν_τ , the lower bounds (2.11) and (2.15) could be far away from the true value of the ^8B neutrino flux. We will obtain now additional inequalities which do not have this drawback. From Eq.(2.9) we have

$$\left\langle \sum_{\ell=e,\mu,\tau} P_{\nu_e \rightarrow \nu_\ell} \right\rangle_{\text{ES}} = \frac{\Sigma^{\text{ES}}}{X_{\nu_\mu e} \Phi_{\text{B}}}. \quad (2.17)$$

Here

$$\Sigma^{\text{ES}} \equiv N^{\text{ES}} - \int_{E_{\text{th}}^{\text{ES}}} (\sigma_{\nu_e e}(E) - \sigma_{\nu_\mu e}(E)) \phi_{\nu_e}(E) dE, \quad (2.18)$$

$$\left\langle \sum_{\ell=e,\mu,\tau} P_{\nu_e \rightarrow \nu_\ell} \right\rangle_{\text{ES}} \equiv \frac{1}{X_{\nu_\mu e}} \int_{E_{\text{th}}^{\text{ES}}} \sigma_{\nu_\mu e}(E) X(E) \sum_{\ell=e,\mu,\tau} P_{\nu_e \rightarrow \nu_\ell}(E) dE . \quad (2.19)$$

and

$$X_{\nu_\mu e} \equiv \int_{E_{\text{th}}^{\text{ES}}} \sigma_{\nu_\mu e}(E) X(E) dE . \quad (2.20)$$

For $E_{\text{th}}^{\text{ES}} \simeq 6 \text{ MeV}$ (which corresponds to a kinetic energy threshold $T_{\text{th}} = 4.5 \text{ MeV}$ for the electrons in the CC process) we have $X_{\nu_\mu e} \simeq 2 \times 10^{-45} \text{ cm}^2$. From Eq.(2.17) we obtain the following lower bound for the total ^8B neutrino flux:

$$\Phi_{\text{B}} \geq \frac{\Sigma^{\text{ES}}}{X_{\nu_\mu e}} . \quad (2.21)$$

The quantity Σ^{ES} can be obtained directly from the data of the SNO and S-K experiments. In fact, N^{ES} will be measured in both experiments. The second term in the right-hand side of Eq.(2.18) can be determined from the CC data of the SNO experiment.

Finally, from Eq.(2.13) we obtain

$$\left\langle \sum_{\ell=e,\mu,\tau} P_{\nu_e \rightarrow \nu_\ell} \right\rangle_{\text{ES};\text{T}} = \frac{\Sigma^{\text{ES}}(\text{T})}{X_{\nu_\mu e}(\text{T}) \Phi_{\text{B}}} . \quad (2.22)$$

Here

$$\Sigma^{\text{ES}}(\text{T}) \equiv n^{\text{ES}}(\text{T}) - \int_{E_{\text{m}}(\text{T})} \left[\frac{d\sigma_{\nu_e e}}{dT}(E, \text{T}) - \frac{d\sigma_{\nu_\mu e}}{dT}(E, \text{T}) \right] \phi_{\nu_e}(E) dE , \quad (2.23)$$

$$\left\langle \sum_{\ell=e,\mu,\tau} P_{\nu_e \rightarrow \nu_\ell} \right\rangle_{\text{ES};\text{T}} \equiv \frac{1}{X_{\nu_\mu e}(\text{T})} \int_{E_{\text{m}}(\text{T})} \frac{d\sigma_{\nu_\mu e}}{dT}(E, \text{T}) X(E) \sum_{\ell=e,\mu,\tau} P_{\nu_e \rightarrow \nu_\ell}(E) dE \quad (2.24)$$

and

$$X_{\nu_\mu e}(\text{T}) \equiv \int_{E_{\text{m}}(\text{T})} \frac{d\sigma_{\nu_\mu e}}{dT}(E, \text{T}) X(E) dE . \quad (2.25)$$

The function $X_{\nu_\mu e}(\text{T})$ is plotted in Fig.1. From Eq.(2.22) we obtain the following lower bound for the total ^8B neutrino flux:

$$\Phi_{\text{B}} \geq \left[\frac{\Sigma^{\text{ES}}(\text{T})}{X_{\nu_\mu e}(\text{T})} \right]_{\text{max}} . \quad (2.26)$$

Thus, we obtained several expressions which give a lower bound for the total flux of ^8B neutrinos (Eqs.(2.4), (2.6), (2.11), (2.15), (2.21), (2.26)). All these expressions depend only on known quantities and quantities that will be measured in the SNO and S-K experiments. When the SNO and S-K data will be available it will be possible to choose the expression

which gives the highest lower bound. This will be the best model independent lower bound for the total flux of ${}^8\text{B}$ neutrinos in the general case of transitions of solar ν_e 's into active and sterile states. We will denote this best lower bound as Φ_B° .

A comparison of Φ_B° with the value of the total flux of ${}^8\text{B}$ neutrinos predicted by the SSM, Φ_B^{SSM} , will be a direct experimental test of the model. If $\Phi_B^\circ \leq \Phi_B^{\text{SSM}}$ no conclusion about the SSM can be reached. If $\Phi_B^\circ > \Phi_B^{\text{SSM}}$ it would mean that the ${}^8\text{B}$ neutrino flux is higher than the flux predicted by the SSM.

In the next section we will use Φ_B° in order to obtain several inequalities whose test will allow to reveal the presence of sterile neutrinos in the flux of solar neutrinos on the earth.

III. STERILE NEUTRINO TESTS

In this section we will obtain several inequalities which could allow to reveal the presence of sterile neutrinos in the flux of solar neutrinos on the earth. Let us consider Eq.(2.1). In the case of transitions of solar ν_e 's only into ν_e , ν_μ and ν_τ we have $\left\langle \sum_{\ell=e,\mu,\tau} P_{\nu_e \rightarrow \nu_\ell} \right\rangle_{\text{NC}} = 1$. In the general case of transitions of ν_e 's into active and sterile states³.

$$\left\langle \sum_{\ell=e,\mu,\tau} P_{\nu_e \rightarrow \nu_\ell} \right\rangle_{\text{NC}} = 1 - \langle P_{\nu_e \rightarrow \nu_s} \rangle_{\text{NC}} < 1. \quad (3.1)$$

Here $P_{\nu_e \rightarrow \nu_s}$ is the probability of transition of solar ν_e 's into all possible sterile states. The right-hand side of Eq.(2.1) depends on the total flux of ${}^8\text{B}$ neutrinos Φ_B . In the case of transitions of solar ν_e 's into sterile states the flux Φ_B cannot be determined from the experimental data. However, as we have seen in the previous section, from the SNO and S-K data it will be possible to determine a lower bound for the total flux of ${}^8\text{B}$ neutrinos, Φ_B° . Therefore, from Eq.(2.1) we obtain

$$\left\langle \sum_{\ell=e,\mu,\tau} P_{\nu_e \rightarrow \nu_\ell} \right\rangle_{\text{NC}} \leq \frac{N^{\text{NC}}}{X_{\nu d}^{\text{NC}} \Phi_B^\circ}. \quad (3.2)$$

The right-hand side of Eq.(3.2) contains only measurable quantities. If it will turn out that $N^{\text{NC}}/X_{\nu d}^{\text{NC}} \Phi_B^\circ < 1$, it will mean that $\nu_e \rightarrow \nu_s$ transitions take place. From Eqs.(3.1) and (3.2), for the averaged value of the probability of transitions of ν_e 's into all possible sterile states $\langle P_{\nu_e \rightarrow \nu_s} \rangle_{\text{NC}}$ we find the following lower bound:

$$\langle P_{\nu_e \rightarrow \nu_s} \rangle_{\text{NC}} \geq 1 - \frac{N^{\text{NC}}}{X_{\nu d}^{\text{NC}} \Phi_B^\circ}. \quad (3.3)$$

³ We assume that neutrinos are stable particles. For a discussion of neutrino instability see Ref. [23] and references therein.

Analogously, from Eqs.(2.17) and (2.22) we obtain the following inequalities for other averages of the probability of transition of solar ν_e 's into all possible sterile states⁴:

$$\langle P_{\nu_e \rightarrow \nu_s} \rangle_{\text{ES}} \geq 1 - \frac{\Sigma^{\text{ES}}}{X_{\nu_\mu e} \Phi_B^\circ} , \quad (3.4)$$

$$\langle P_{\nu_e \rightarrow \nu_s} \rangle_{\text{ES};\text{T}} \geq 1 - \frac{\Sigma^{\text{ES}}(\text{T})}{X_{\nu_\mu e}(\text{T}) \Phi_B^\circ} . \quad (3.5)$$

If it will turn out that the right-hand side of at least one of these inequalities is different from zero, it will mean that active neutrinos transform into sterile states and that the probability of $\nu_e \rightarrow \nu_s$ transitions depends on the neutrino energy E . Let us stress again that a discovery of such transitions will have a great impact on the theory.

IV. TRANSITIONS OF SOLAR ν_e 'S ONLY INTO STERILE STATES

If the tests proposed in the previous section will demonstrate the presence of $\nu_e \rightarrow \nu_s$ transitions, the question will arise whether there are also transitions of solar ν_e 's into active ν_μ and/or ν_τ . When solar neutrinos will be detected through the observation of CC, NC and ES processes, it will be possible to reveal the presence of ν_μ and/or ν_τ in the flux of solar neutrinos on the earth independently from $\nu_e \rightarrow \nu_s$ transitions [24]. In fact, let us consider the quantities

$$R^{\text{ES}} = 1 - \frac{1}{N^{\text{ES}}} \int_{E_{\text{th}}^{\text{ES}}} \sigma_{\nu_e e}(E) \phi_{\nu_e}(E) dE , \quad (4.1)$$

$$R^{\text{NC}} = 1 - \frac{1}{N^{\text{NC}}} \int_{E_{\text{th}}^{\text{NC}}} \sigma_{\nu_d}^{\text{NC}}(E) \phi_{\nu_e}(E) dE , \quad (4.2)$$

which give the relative contribution of ν_μ and ν_τ to the ES and NC event rates, respectively. Here $\phi_{\nu_e}(E)$ is the flux of solar ν_e 's on the earth, which will be determined for $E \geq E_{\text{th}}^{\text{CC}} \simeq 6$ MeV from the CC data of the SNO experiment. Since in both the SNO and S-K experiment it is possible to choose an energy threshold for the ES process $E_{\text{th}}^{\text{ES}} \geq 6$ MeV, the ratio R^{ES} will be determined directly from the experimental data. On the other hand, the NC threshold in the SNO experiment is fixed at $E_{\text{th}}^{\text{NC}} = 2.2$ MeV, but there will be no direct experimental information on the value of $\phi_{\nu_e}(E)$ in the energy interval $E_{\text{th}}^{\text{NC}} \leq E \leq E_{\text{th}}^{\text{CC}}$. It is obvious that

$$R^{\text{NC}} \leq 1 - \frac{1}{N^{\text{NC}}} \int_{E_{\text{th}}^{\text{CC}}} \sigma_{\nu_d}^{\text{NC}}(E) \phi_{\nu_e}(E) dE . \quad (4.3)$$

⁴ It is easy to see that if $P_{\nu_e \rightarrow \nu_s}(E) = \text{const}$ the ratios in the right-hand sides of Eqs.(3.3)–(3.5) are larger or equal to one. Therefore, in this case one cannot reach any conclusion about transitions of solar ν_e 's into sterile states.

Let us notice that the contribution of the integral $\frac{1}{N^{\text{NC}}} \int_{E_{\text{th}}^{\text{NC}}}^{E_{\text{th}}^{\text{CC}}} \sigma_{\nu d}^{\text{NC}}(E) \phi_{\nu_e}(E) dE$ to the right-hand part of Eq.(4.2) is expected to be small due to the smallness of the cross section in the corresponding energy interval. Our model calculations show that this contribution is of the order of a few percentage.

In the following we assume that from the ratio R^{ES} and the upper bound (4.3) for R^{NC} no indication in favor of the presence of ν_μ and/or ν_τ in the solar neutrino flux on the earth will be found. If, in addition, it will be found that the spectrum of solar ν_e 's is distorted (i.e. the quantity $\phi_{\nu_e}(E)/X(E)$ is energy dependent), it will mean that solar ν_e 's transform only into sterile states. These transitions could take place if ν_e is mixed only with sterile neutrinos (as in the models presented in Ref. [25]) or if neutrinos have large Dirac magnetic moments ($\sim 10^{-11}$ – $10^{-10} \mu_B$) and left-handed solar ν_e 's transform partly into right-handed sterile states due to precessions in the magnetic field of the sun [26]. As it is well known, in the last case the flux of solar ν_e 's has a time dependence anti-correlated with the solar activity.

If solar neutrinos are transformed only into sterile states, the best lower bound for the total ^8B neutrino flux is given by Eq.(2.6). In fact, in this case $[P_{\nu_e \rightarrow \nu_e}]_{\text{max}} \geq \langle P_{\nu_e \rightarrow \nu_e} \rangle_a$ (with $a = \text{NC}, \text{ES}, \dots$). From this inequality it follows that

$$\Phi_B^\circ = \left[\frac{\phi_{\nu_e}(E)}{X(E)} \right]_{\text{max}}. \quad (4.4)$$

Let us discuss now what informations about the probability of ν_e 's to survive can be obtained from the experimental data in the case under consideration. From the measurement of the charged current spectrum the probability $P_{\nu_e \rightarrow \nu_e}(E)$ can be determined up to a constant (see Eq.(2.5)). From Eq.(4.4) we obtain the following upper bound for the survival probability:

$$P_{\nu_e \rightarrow \nu_e}(E) \leq \frac{\phi_{\nu_e}(E)/X(E)}{[\phi_{\nu_e}(E)/X(E)]_{\text{max}}}. \quad (4.5)$$

Furthermore, from the NC event rate we obtain the following upper bound:

$$\langle P_{\nu_e \rightarrow \nu_e} \rangle_{\text{NC}} \leq \frac{N^{\text{NC}}}{X_{\nu d}^{\text{NC}} \Phi_B^\circ}. \quad (4.6)$$

Additional informations about the probability $P_{\nu_e \rightarrow \nu_e}$ can be obtained from the measurement of the ES event rate and ES recoil spectrum. Let us determine the average value:

$$\langle P_{\nu_e \rightarrow \nu_e} \rangle_{\nu_{ee}} \equiv \frac{1}{X_{\nu_{ee}}} \int_{E_{\text{th}}^{\text{ES}}} \sigma_{\nu_{ee}}(E) X(E) P_{\nu_e \rightarrow \nu_e}(E) dE = \frac{N^{\text{ES}}}{X_{\nu_{ee}} \Phi_B}, \quad (4.7)$$

We obtain

$$\langle P_{\nu_e \rightarrow \nu_e} \rangle_{\nu_{ee}} \leq \frac{N^{\text{ES}}}{X_{\nu_{ee}} \Phi_B^\circ}, \quad (4.8)$$

Finally, we have

$$\langle P_{\nu_e \rightarrow \nu_e} \rangle_{\nu_e e; T} \equiv \frac{1}{X_{\nu_e e}(T)} \int_{E_m(T)} \frac{d\sigma_{\nu_e e}}{dT}(E, T) X(E) P_{\nu_e \rightarrow \nu_e}(E) dE = \frac{n^{\text{ES}}(T)}{X_{\nu_e e}(T) \Phi_B} . \quad (4.9)$$

From Eq.(4.9) it follows that

$$\langle P_{\nu_e \rightarrow \nu_e} \rangle_{\nu_e e; T} \leq \frac{n^{\text{ES}}(T)}{X_{\nu_e e}(T) \Phi_B^{\circ}} . \quad (4.10)$$

Thus, in the case under consideration we can obtain informations about the probability of ν_e 's to survive not only from the CC data, but also from the ES and NC data.

Finally, we will present the results of a calculation of some of the lower bounds for the $\nu_e \rightarrow \nu_s$ transition probability in a simple model with ν_e - ν_s mixing which, as was mentioned in the introduction, can explain the existing solar neutrino data. The values of the parameters of the model are given in Eq.(1.4). In our calculations we assumed the SSM ^8B neutrino flux in order to estimate the number of CC and NC events in the SNO experiment and the number of ES events in the S-K experiment after one year of data taking. From these numbers we estimated the statistical accuracy with which the calculated quantities will be measured. We obtained $R^{\text{ES}} = 0.00 \pm 0.02$ and $R^{\text{NC}} \leq 0.03 \pm 0.03$. Therefore, it will be possible to establish the absence of $\nu_e \rightarrow \nu_{\mu(\tau)}$ transitions with a good accuracy. For the lower bound of the total flux of ^8B neutrinos, from Eq.(2.6) we found $\Phi_B \geq (3.0 \pm 0.2) \times 10^6 \text{ cm}^{-2} \text{ sec}^{-1}$. Using this result, we obtained $\langle P_{\nu_e \rightarrow \nu_s} \rangle_{\text{NC}} \geq 0.24 \pm 0.04$ and $\langle P_{\nu_e \rightarrow \nu_s} \rangle_{\text{CC}} \geq 0.22 \pm 0.04$.

The results of our calculations for the lower bounds (4.8), (4.10) and (4.5) are presented in Fig.2, Fig.3 and Fig.4. The solid line in Fig.2 represent the lower bound for $\langle P_{\nu_e \rightarrow \nu_s} \rangle_{\nu_e e}$ as a function of the kinetic energy threshold T_{th} of the recoil electrons in the ES process. The dotted lines represent the 1σ statistical errors after one year of data taking. The dash-dotted line represent the value of $\langle P_{\nu_e \rightarrow \nu_s} \rangle_{\nu_e e}$ in the model. It can be seen from Fig.2 that the lower bound for $\langle P_{\nu_e \rightarrow \nu_s} \rangle_{\nu_e e}$ is bigger for a lower energy threshold. The solid lines in Fig.3 and Fig.4 represent the results of the calculations of the lower bounds for $\langle P_{\nu_e \rightarrow \nu_s} \rangle_{\nu_e e; T}$ and $P_{\nu_e \rightarrow \nu_s}(E)$, respectively. The error-bars represent our estimation of the 1σ statistical accuracy with which these lower bounds could be determined after one year of data taking. The dash-dotted lines represent the behaviour of the probabilities $\langle P_{\nu_e \rightarrow \nu_s} \rangle_{\nu_e e; T}$ and $P_{\nu_e \rightarrow \nu_s}(E)$ in the model.

Our estimations illustrate that the future SNO and S-K experiments have reasonable possibilities to reveal transitions of solar neutrinos into sterile states.

V. CONCLUSIONS

A discovery of transitions of solar ν_e 's into sterile states would be a direct discovery of new physics beyond the Standard Model. We showed here that the future solar neutrino experiments SNO and S-K, in which neutrinos from ^8B decay will be detected, may allow to reveal the presence of $\nu_e \rightarrow \nu_s$ transitions and to obtain lower bounds for averaged values of the probability $P_{\nu_e \rightarrow \nu_s}$. We showed also that it will be possible to obtain from the data

of the SNO and S-K experiments a model independent lower bound for the total flux of ^8B neutrinos in the general case of transitions of solar ν_e 's into active as well as into sterile neutrinos.

In the derivation of the expressions of the lower bounds for the total flux of ^8B neutrinos and for the averages of the probability $P_{\nu_e \rightarrow \nu_s}$ we took into account that in the SNO experiment solar neutrinos will be detected through the observation of the CC and ES processes (1.5) and (1.7) as well as the pure NC process (1.6). In the ICARUS experiment [27] the high energy ^8B neutrinos will be detected through the observation of the CC process $\nu_e + {}^{40}\text{Ar} \rightarrow e^- + {}^{40}\text{K}^*$ and the ES process (1.7). A detailed investigation of the CC electron spectrum will be carried out. The ICARUS experiment can also reveal the presence of sterile neutrinos in the solar neutrino flux on the earth in a model independent way. In the case of the ICARUS experiment, a lower bound for the total flux of ^8B neutrinos can be obtained from the relations (2.6), (2.7), (2.11), (2.15), (2.21) and (2.26) and lower bounds for $\langle P_{\nu_e \rightarrow \nu_s} \rangle_{\text{ES}}$ and $\langle P_{\nu_e \rightarrow \nu_s} \rangle_{\text{ES;T}}$ can be obtained from the relations (3.4) and (3.5). It is necessary, however, to stress that the contribution of the total probability $\sum_{\ell=e,\mu,\tau} P_{\nu_e \rightarrow \nu_\ell}$ to

the ES event rate is suppressed by the smallness of the ratio $\sigma_{\nu_\mu e}/\sigma_{\nu_e e}$. In order to obtain from the ES and CC processes informations about the $\nu_e \rightarrow \nu_s$ transitions it is necessary to have high statistics CC and ES data. The ICARUS experiment is assumed to have this characteristic.

REFERENCES

- [1] B. Pontecorvo, JETP (Sov. Fiz.) **53**, 1717 (1967).
- [2] R. Davis Jr., *Proc. of the 6th International Workshop on Neutrino Telescopes*, Venezia, March 1994, edited by M. Baldo Ceolin.
- [3] K. S. Hirata et al., Phys. Rev. Lett. **65**, 1297 (1990); Phys. Rev. D **44**, 2241 (1991); Y. Suzuki, *Proc. of the 6th International Workshop on Neutrino Telescopes*, Venezia, March 1994, edited by M. Baldo Ceolin.
- [4] GALLEX Coll., Phys. Lett. B **285**, 376 (1992); Phys. Lett. B **314**, 445 (1993); Phys. Lett. B **327**, 377 (1994); Phys. Lett. B **342**, 440 (1995).
- [5] J.N. Abdurashitov et al., Phys. Lett. B **328**, 234 (1994); Talk presented at the 27th *Int. Conf. on High Energy Physics*, Glasgow, July 1994.
- [6] J.N. Bahcall and R. Ulrich, Rev. Mod. Phys. **60**, 297 (1988); J.N. Bahcall, *Neutrino Physics and Astrophysics*, Cambridge University Press, 1989; J.N. Bahcall and M.H. Pinsonneault, Rev. Mod. Phys. **64**, 885 (1992).
- [7] S. Turck-Chièze, S. Cahen, M. Cassé and C. Doom, Astrophys. J. **335**, 415 (1988); S. Turck-Chièze and I. Lopes, Astrophys. J. **408**, 347 (1993); S. Turck-Chièze et al., Phys. Rep. **230**, 57 (1993).
- [8] V. Castellani, S. Degl’Innocenti and G. Fiorentini, Astronomy & Astrophysics **271**, 601 (1993); S. Degl’Innocenti, Univ. of Ferrara preprint INFN-FE-07-93.
- [9] J.N. Bahcall and H.A. Bethe, Phys. Rev. Lett. **65**, 2233 (1990); Phys. Rev. D **47**, 1298 (1993); V. Castellani, S. Degl’Innocenti and G. Fiorentini, Astronomy & Astrophysics **271**, 601 (1993); V. Castellani et al., Phys. Lett. B **324**, 425 (1994); Phys. Rev. D **50**, 4749 (1994); S. Degl’Innocenti, G. Fiorentini and M. Lissia, Univ. of Ferrara preprint INFN-FE-10-94 (hep-ph@xxx.lanl.gov/9408386); S.A. Bludman, N. Hata and P.G. Langacker, Phys. Rev. D **49**, 3622 (1994); V. Berezinsky, *Proc. of the 6th International Workshop on Neutrino Telescopes*, Venezia, March 1994, edited by M. Baldo Ceolin; J.N. Bahcall; Phys. Lett. B **338**, 276 (1994).
- [10] GALLEX Coll., Phys. Lett. B **285**, 390 (1992); X. Shi, D.N. Schramm and J.N. Bahcall, Phys. Rev. Lett. **69**, 717 (1992); S.A. Bludman, N. Hata, D.C. Kennedy and P.G. Langacker, Phys. Rev. D **47**, 2220 (1993); N. Hata and P.G. Langacker, Phys. Rev. **50**, 632 (1994); P.I. Krastev and S.T. Petcov, Phys. Lett. B **299**, 99 (1993); preprint SISSA-41/94/EP (hep-ph@xxx.lanl.gov/9408234); L.M. Krauss, E. Gates and M. White, Phys. Lett. B **299**, 94 (1993); Fermilab-Pub-94/176-A (hep-ph@xxx.lanl.gov/9406396); G.L. Fogli and E. Lisi, Astropart. Phys. **2**, 91 (1994); G. Fiorentini et al., Phys. Rev. D **49**, 6298 (1994).
- [11] S.P. Mikheyev and A.Yu. Smirnov, Yad. Fiz. **42**, 1441 (1985) [Sov. J. Nucl. Phys. **42**, 913 (1985)]; Il Nuovo Cimento C **9**, 17 (1986); L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978); Phys. Rev. D **20**, 2634 (1979).
- [12] V. Barger, R.J.N. Phillips, and K. Whisnant, Phys. Rev. Lett. **69**, 3135 (1992); P.I. Krastev and S.T. Petcov, Phys. Lett. B **299**, 99 (1993); Phys. Rev. Lett. **72**, 1960 (1994); preprint SISSA-41/94/EP (hep-ph@xxx.lanl.gov/9408234).
- [13] S.A. Bludman, N. Hata, D.C. Kennedy and P.G. Langacker, Phys. Rev. D **47**, 2220 (1993); N. Hata and P.G. Langacker, Phys. Rev. **50**, 632 (1994); P.I. Krastev and S.T. Petcov, preprint SISSA-41/94/EP (hep-ph@xxx.lanl.gov/9408234).

- [14] S.M. Bilenky and S.T. Petcov, Rev. Mod. Phys. **59**, 671 (1987).
- [15] K. Enqvist, K. Kainulainen and J. Maalampi, Phys. Lett. B **249**, 531 (1990); K. Enqvist, K. Kainulainen and M. Thompson, Nucl. Phys. B **373**, 498 (1992); X. Shi, D.N. Schramm and B.D. Fields, Phys. Rev. **48**, 2563 (1993);
- [16] P.G. Langacker, Lectures presented at TASI-90, Boulder, June 1990 (Pennsylvania Univ. preprint UPR-0470T).
- [17] S.M. Bilenky and C. Giunti, Phys. Lett. B **320**, 323 (1994).
- [18] SNO Coll., Phys. Lett. B **194**, 321 (1987); H.H. Chen, Nucl. Instr. Meth. A **264**, 48 (1988); G.T. Ewan, *Proc. of the 4th International Workshop on Neutrino Telescopes*, Venezia, March 1992, edited by M. Baldo Ceolin.
- [19] A. Suzuki, Talk presented at the *Workshop on Elementary Particle Picture of the Universe*, Tsukuba, Japan, 1987 (Tokyo Univ. preprint UT-ICEPP-87-06); Y. Totsuka, Tokyo Univ. preprint ICRR-227-90-20 (1990); C.B. Bratton et al., *Proposal to Participate in the Super-Kamiokande Project*, Dec. 1992.
- [20] J.N. Bahcall, Phys. Rev. D **44**, 1644 (1991).
- [21] J.N. Bahcall and B.R. Holstein, Phys. Rev. C **33**, 2121 (1986).
- [22] K. Kubodera and S. Nozawa, Int. J. Mod. Phys. E **3**, 101 (1994).
- [23] Z. Berezhiani, M. Moretti and A. Rossi, Z. Phys. C **58**, 423 (1993); Z. Berezhiani and A. Rossi, Nucl. Phys. B (Proc. Suppl.) **35**, 469 (1994).
- [24] S.M. Bilenky and C. Giunti, Phys. Lett. B **311**, 179 (1993); Nucl. Phys. (Proc. Suppl.) **35**, 430 (1994).
- [25] D.O. Caldwell and R.N. Mohapatra, Phys. Rev. D **48**, 3259 (1993); Phys. Rev. D **50**, 3477 (1994); J.T. Peltoniemi and J.W.F. Valle, Nucl. Phys. B **406**, 409 (1993).
- [26] A. Cisneros, Astrophys. Space. Sci. **10**, 87 (1970); M.B. Voloshin, M.I. Vysotsky and L.B. Okun, Zh. Eksp. Teor. Fiz. **91**, 754 (1986) [Sov. Phys. JETP **64**, 446 (1986)]; C.S. Lim and W.J. Marciano, Phys. Rev. D **37**, 1368 (1988); E.Kh. Akhmedov, Phys. Lett. B **213**, 64 (1988).
- [27] J.N. Bahcall et al., Phys. Lett. B **178**, 324 (1986); ICARUS Coll., Gran Sasso Lab. preprint LNGS-94/99-I, May 1994.

FIGURES

FIG. 1. Plot of the functions $X_{\nu_\mu e}(T)$ and $X_{\nu_e e}(T)$ defined in Eqs.(2.25) and (2.16), respectively. The depicted range for the kinetic energy T of the recoil electrons in the ES process will be explored by SNO with $T_{\text{th}} = 4.5 \text{ MeV}$.

FIG. 2. Plot of the lower bound for $\langle P_{\nu_e \rightarrow \nu_s} \rangle_{\nu_e e}$ as a function of the kinetic energy threshold T_{th} of the recoil electrons in the ES process calculated in a model with $\nu_e - \nu_s$ mixing (solid line). The dotted lines are our estimation of the 1σ statistical accuracy with which this lower bound will be determined after one year of data taking. The dash-dotted line represent the value of $\langle P_{\nu_e \rightarrow \nu_s} \rangle_{\nu_e e}$ calculated in the model.

FIG. 3. Plot of the lower bound for $\langle P_{\nu_e \rightarrow \nu_s} \rangle_{\nu_e e; T}$ as a function of the kinetic energy T of the recoil electrons in the ES process calculated in a model with $\nu_e - \nu_s$ mixing (solid line). The 1σ error-bars are our estimation of the statistical accuracy with which this lower bound will be determined after one year of data taking. The dash-dotted line represent the value of $\langle P_{\nu_e \rightarrow \nu_s} \rangle_{\nu_e e; T}$ calculated in the model.

FIG. 4. Plot of the lower bound for the probability $P_{\nu_e \rightarrow \nu_s}(E)$ as a function of the neutrino energy E calculated in a model with $\nu_e - \nu_s$ mixing (solid line). The 1σ error-bars are our estimation of the statistical accuracy with which this lower bound will be determined after one year of data taking. The dash-dotted line represent the value of $P_{\nu_e \rightarrow \nu_s}(E)$ calculated in the model.

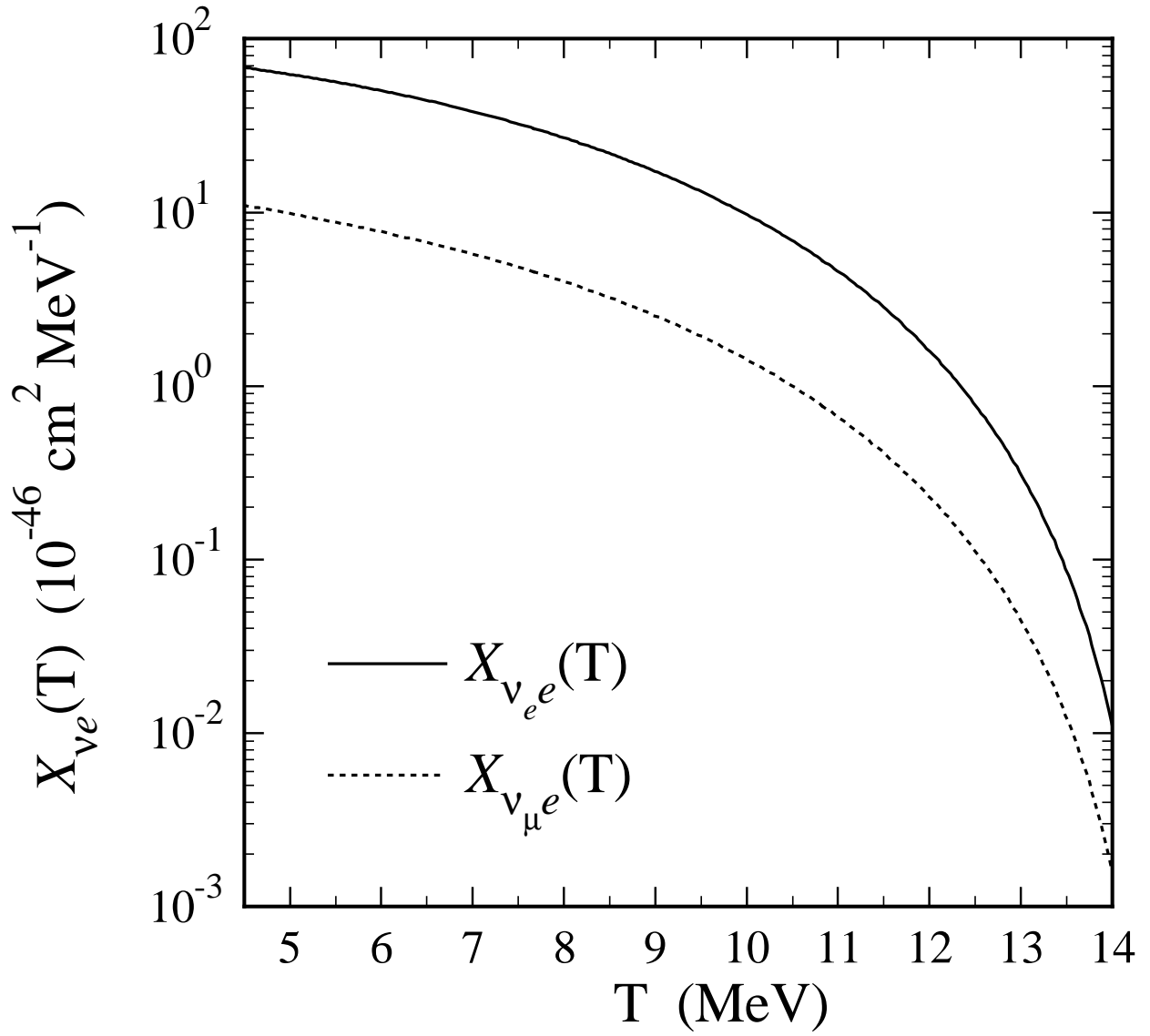


FIG. 1. Plot of the functions $X_{\nu_{\mu} e}(T)$ and $X_{\nu_e e}(T)$ defined in Eqs.(2.25) and (2.16), respectively. The depicted range for the kinetic energy T of the recoil electrons in the ES process will be explored by SNO with $T_{\text{th}} = 4.5$ MeV.

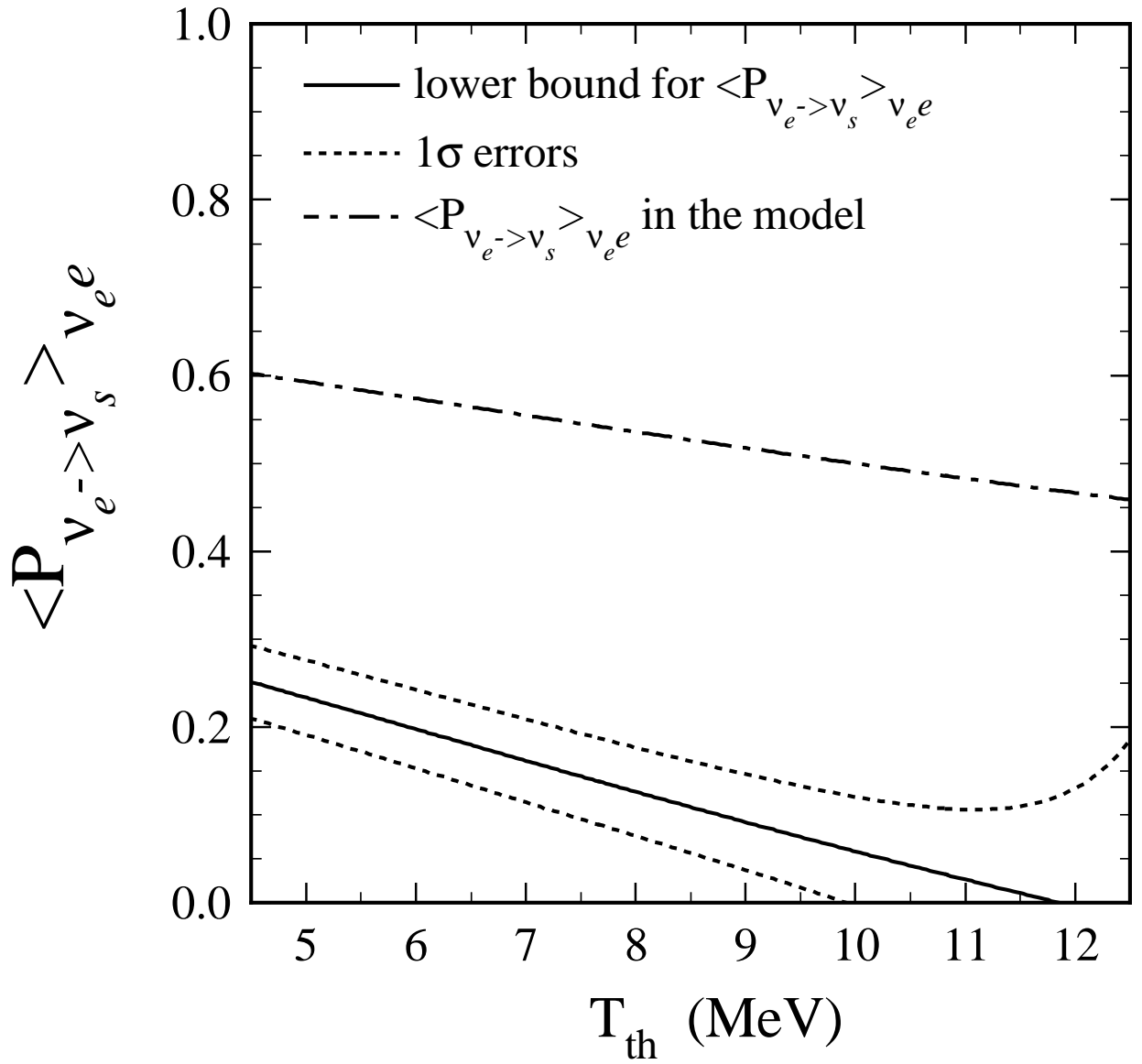


FIG. 2. Plot of the lower bound for $\langle P_{\nu_e \rightarrow \nu_s} \rangle_{\nu_e e}$ as a function of the kinetic energy threshold T_{th} of the recoil electrons in the ES process calculated in a model with ν_e - ν_s mixing (solid line). The dotted lines are our estimation of the 1σ statistical accuracy with which this lower bound will be determined after one year of data taking. The dash-dotted line represent the value of $\langle P_{\nu_e \rightarrow \nu_s} \rangle_{\nu_e e}$ calculated in the model.

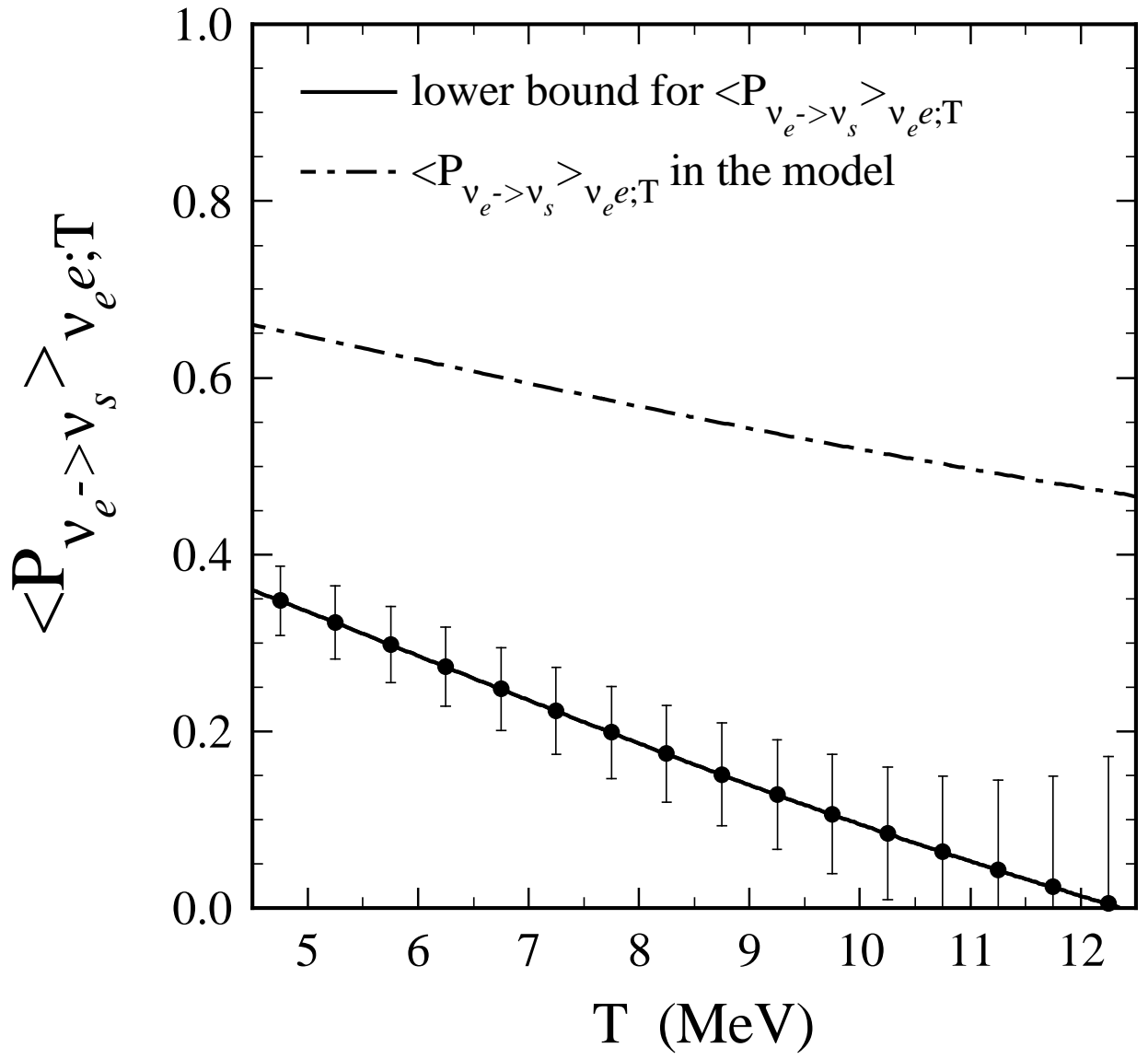


FIG. 3. Plot of the lower bound for $\langle P_{\nu_e \rightarrow \nu_s} \rangle_{\nu_e e; T}$ as a function of the kinetic energy T of the recoil electrons in the ES process calculated in a model with $\nu_e - \nu_s$ mixing (solid line). The 1σ error-bars are our estimation of the statistical accuracy with which this lower bound will be determined after one year of data taking. The dash-dotted line represent the value of $\langle P_{\nu_e \rightarrow \nu_s} \rangle_{\nu_e e; T}$ calculated in the model.

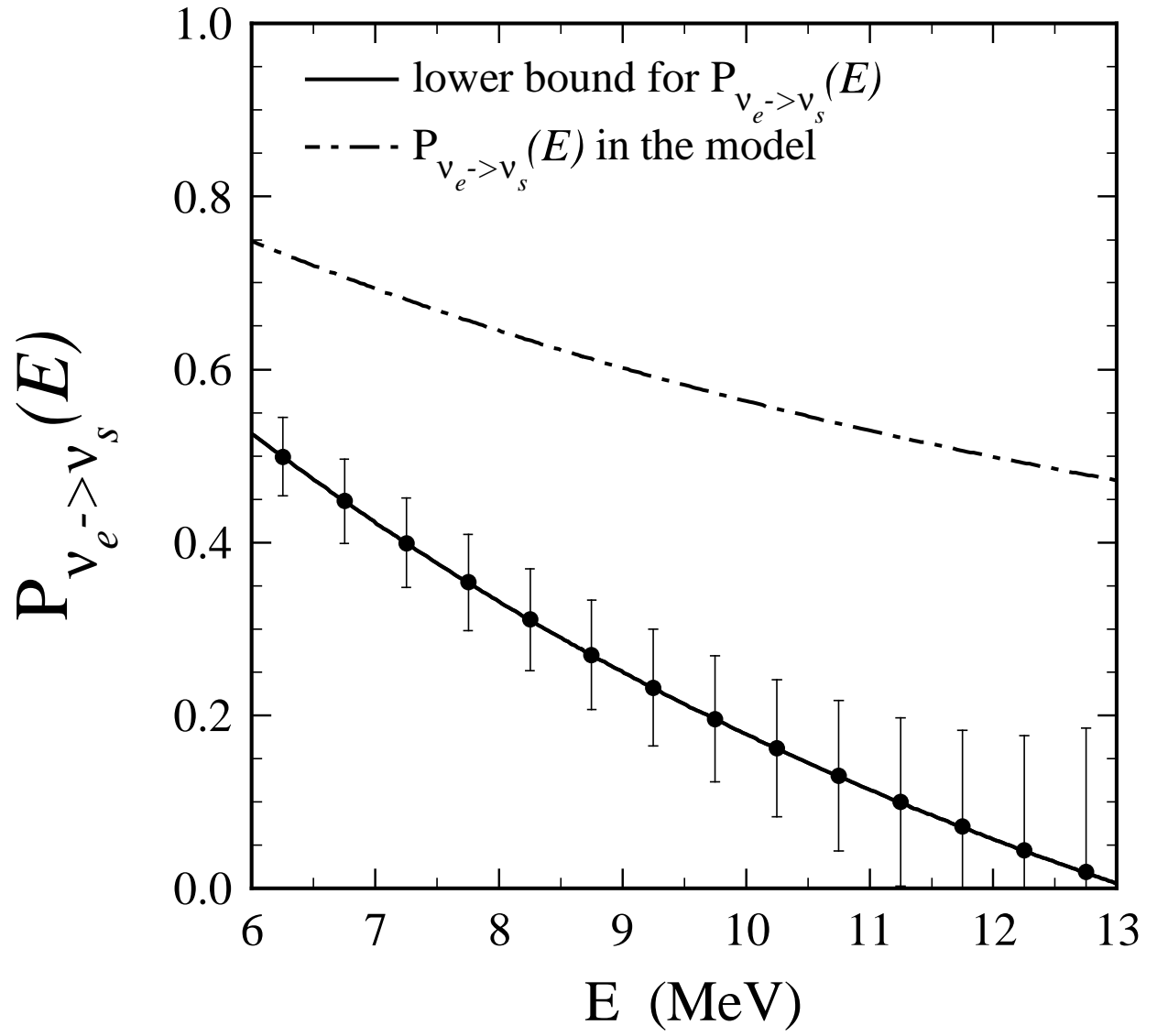


FIG. 4. Plot of the lower bound for the probability $P_{\nu_e \rightarrow \nu_s}(E)$ as a function of the neutrino energy E calculated in a model with ν_e - ν_s mixing (solid line). The 1σ error-bars are our estimation of the statistical accuracy with which this lower bound will be determined after one year of data taking. The dash-dotted line represent the value of $P_{\nu_e \rightarrow \nu_s}(E)$ calculated in the model.

This figure "fig1-1.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9502263v2>

This figure "fig1-2.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9502263v2>

This figure "fig1-3.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9502263v2>

This figure "fig1-4.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9502263v2>